# **DEPARTMENT OF MATHEMATICS**

# DETAILED COURSE OUTLINE OF MATHEMATICS BS (4 YEARS PROGRAM)

Course Name: Calculus-I	Course Code:MBS-311
Course Structure: Lectures: 4	<b>Credit Hours:</b> 4+0
Prerequisites: Knowledge ofIntermediate Calculus	

## **Specific Objectives of course:**

Calculus serves as the foundation of advanced subjects in all areas of mathematics. This is the first course of Calculus. The objective of this course is to introduce students to the fundamental concepts of limit, continuity, differential and integral calculus of functions of one variable.

## **Course Outline:**

**Equations and inequalities:** Solving linear and quadratic equations, linear inequalities. Division of polynomials, synthetic division. Roots of polynomial, rational roots; Viete Relations. Descartes rule of signs. Solutions of equations with absolute value sign. Solution of linear and non-linear inequalities with absolute value sign.

**Functions and graphs:** Domain and range of a function. Examples: polynomial, rational, piecewise defined functions, absolute value functions, and evaluation of such functions. Operations with functions: sum, product, quotient and composition. Graphs of functions: linear, quadratic, piecewise defined functions.

**Lines and systems of equations:** Equation of a straight line, slope and intercept of a line, parallel and perpendicular lines. Systems of linear equations, solution of system of linear equations. Nonlinear systems: at least one quadratic equation.

**Limits and continuity:** Functions, limit of a function. Graphical approach. Properties of limits. Theorems of limits. Limits of polynomials, rational and transcendental functions. Limits at infinity, infinite limits, one-sided limits. Continuity.

**Derivatives:** Definition, techniques of differentiation. Derivatives of polynomials and rational, exponential, logarithmic and trigonometric functions. The chain rule. Implicit differentiation. Rates of change in natural and social sciences. Related rates. Linear approximations and differentials. Higher derivatives, Leibnitz's theorem.

**Applications of derivatives:** Increasing and decreasing functions. Relative extrema and optimization. First derivative test for relative extrema. Convexity and

point of inflection. The second derivative test for extrema. Curve sketching. Mean value theorems. Indeterminate forms and L'Hopitals rule. Inverse functions and their derivatives.

**Integration**: Anti derivatives and integrals. Riemann sums and the definite integral. Properties of Integral. The fundamental theorem of calculus. The substitution rule.

## **Recommended Books:**

- 1. Thomas, *Calculus*, 11<sup>th</sup> Edition. Addison Wesley Publishing Company, 2005.
- H. Anton, I. Bevens, S. Davis, *Calculus*, 8<sup>th</sup> Edition, John Wiley & Sons, Inc. 2005
- 3. Hughes-Hallett, Gleason, McCallum, et al, *Calculus Single and Multivariable*, 3<sup>rd</sup> Edition. John Wiley & Sons, Inc. 2002.
- 4. Frank A. Jr, Elliott Mendelson, *Calculus*, Schaum's outlines series, 4<sup>th</sup> Edition, 1999
- 5. C.H. Edward and E.D Penney, *Calculus and Analytics Geometry*, Prentice Hall, Inc. 1988
- 6. E. W. Swokowski, *Calculus with Analytic Geometry*, PWS Publishers, Boston, Massachusetts, 1983.
- 7. M. Liebeck, *A Concise introduction to pure Mathematics*, CRC Press, 2011.
- 8. A. Kaseberg, Intermediate Algebra, Thomson Brooks/cole, 2004.

Course Name: Title of the Course: Elements of Set	Course Code: MBS-312	
Theory and Mathematical Logic		
Course Structure: Lectures: 3	<b>Credit Hours:</b> 3	
Prerequisites: Knowledge of Intermediate Mathematics		
Specific Objectives of course:		
Everything mathematicians do can be reduced to statements about sets, equality		
and membership which are basics of set theory. This course introduces these basic		
concepts. The course aims at familiarizing the students with cardinals, relations		
and fundamentals of propositional and predicate logics.		
Course Outline:		
Set theory: Sets, subsets, operations with sets: union, intersection, difference,		
symmetric difference, Cartesian product and disjoint union. Functions: graph of a		

function. Composition; injections, surjections, bijections, inverse function.

**Computing cardinals:** Cardinality of Cartesian product, union. Cardinality of all functions from a set to another set. Cardinality of all injective, surjective and bijective functions from a set to another set. Infinite sets, finite sets. Countable sets, properties, examples (Z, Q). R is not countable. R, RxR, RxRxR have the same cardinal. Operations with cardinal numbers. Cantor-Bernstein theorem. **Relations:** Equivalence relations, partitions, quotient set; examples, parallelism, similarity of triangles. Order relations, min, max, inf, sup; linear order. Examples: N, Z, R, P(A). Well ordered sets and induction. Inductively ordered sets and Zorn's lemma.

# Mathematical logic:

Propositional Calculus. Truth tables. Predicate Calculus.

- 1. M. Liebeck, A Concise Introduction to Pure Mathematics, CRC Press, 2011.
- 2. 2. N. L. Biggs, *Discrete Mathematics*, Oxford University Press, 2002.
- 3. R. Garnier, J. Taylor, *Discrete Mathematics*, Chapters 1,3,4,5, CRC Press, 2010.
- 4. A. A. Fraenkal, *Abstract Set Theory*, North-Holland Publishing Company, 1966.
- 5. P. Suppes, Axiomatic Set Theory, Dover Publication, 1972.
- 6. P.R. Halmos, *Naive Set Theory*, New York, Van Nostrand, 1950.
- 7. B. Rotman, G.T. Kneebone, *TheTheory of sets and Transfinite Numbers*, Oldbourne London, 1968.
- 8. D. Smith, M. Eggen, R.St. Andre, *A Transition to Advanced Mathematics*, Brooks/Cole, 2001.

Course Name: Calculus-III	Course Code:
Course Structure: Lectures: 3	Credit Hours: 04
Prerequisites: Calculus-II	
<b>Specific Objectives of course</b> This is third course of (	Palculus and builds up on the
concepts learned in first two courses. The students wou	ld be introduced to the vector
calculus, the calculus of multivariable functions and do with their applications	ouble and triple integrals along
Course Outline:	

**Vectors and analytic geometry in space**: Coordinate system. Rectangular, cylindrical and spherical coordinates. The dot product, the cross product. Equations of lines and planes. Quadric surfaces.

**Vector-valued functions**: Vector-valued functions and space curves. Derivatives and integrals of vector valued functions. Arc length. Curvature, normal and binormal vectors. **Multivariable functions and partial derivatives**: Functions of several variables. Limits and Continuity. Partial derivatives, Composition and chain rule. Directional derivatives and the gradient vector. Implicit function theorem for several variables. Maximum and minimum values. Optimization problems. Lagrange Multipliers.

**Multiple integrals**: Double integrals over rectangular domains and iterated integrals. Non-rectangular domains. Double integrals in polar coordinates. Triple integrals in rectangular, cylindrical and spherical coordinates. Applications of double and triple integrals. Change of variables in multiple integrals.

**Vector calculus**: Vector fields. Line integrals. Green's theorem. Curl and divergence. Surface integrals over scalar and vector fields. Divergence theorem. Stokes' theorem.

- 1. Thomas, *Calculus*, 11<sup>th</sup> Edition. Addison Wesley Publishing Company, 2005
- 2. H. Anton, I. Bevens, S. Davis, *Calculus*, 8<sup>th</sup> Edition, John Wiley & Sons, Inc. 2005
- 3. Hughes-Hallett, Gleason, McCallum, et al, *Calculus Single and Multivariable*, 3<sup>rd</sup> Edition. John Wiley & Sons, Inc. 2002.
- 4. Frank A. Jr, Elliott Mendelson, Calculus, Schaum's outlines series, 4th Edition, 1999
- 5. C.H. Edward and E.D Penney, *Calculus and Analytics Geometry*, Prentice Hall, Inc. 1988
- 6. E. W. Swokowski, *Calculus with Analytic Geometry*, PWS Publishers, Boston, Massachusetts, 1983.
- 7. M. Liebeck, A Concise introduction to pure Mathematics, CRC Press, 2011.
- 8. A. Kaseberg, Intermediate Algebra, Thomson Brooks/COLE, 2004.
- 9. J. Stewart, *Calculus early transcendentals*, 7<sup>th</sup> Edition, Brooks/COLE, 2008.

Course Name: Calculus-II	Course Code: MBS- 321
Course Structure: Lectures: 3	Credit Hours: 03
Prerequisites: Calculus-I	

This is second course of Calculus. As continuation of Calculus I, it focuses on techniques of integration and applications of integrals. The course also aims at introducing the students to infinite series, parametric curves and polar coordinates.

#### **Course Outline:**

**Techniques of integration**: Integrals of elementary, hyperbolic, trigonometric, logarithmic and exponential functions. Integration by parts, substitution and partial fractions. Approximate integration. Improper integrals. Gamma functions.

**Applications of integrals:** Area between curves, average value, Volumes, Arc length. Area of a surface of revolution. Applications to Economics, Physics, Engineering and Biology.

**Infinite series**: Sequences and series. Convergence and absolute convergence. Tests for convergence: divergence test, integral test, pseries test, comparison test, limit comparison test, alternating series test, ratio test, root test. Power series. Convergence of power series. Representation of functions as power series. Differentiation and integration of power series. Taylor and McLaurin series.

Approximations by Taylor polynomials.

**Conic section, parameterized curves and polar coordinates**: Curves defined by parametric equations. Calculus with parametric curves: tangents, areas, arc length. Polar coordinates. Polar curves, tangents to polar curves. Areas and arc length in polar coordinates

- 1. Thomas, Calculus, 11th Edition. Addison Wesley Publishing Company, 2005
- H. Anton, I. Bevens, S. Davis, *Calculus*, 8<sup>th</sup> Edition, John Wiley & Sons, Inc. 2005
- 3. Hughes-Hallett, Gleason, McCallum, et al, *Calculus Single and Multivariable*, 3<sup>rd</sup> Edition. John Wiley & Sons, Inc. 2002.
- 4. Frank A. Jr, Elliott Mendelson, *Calculus*, Schaum's outlines series, 4<sup>th</sup> Edition, 1999
- 5. C.H. Edward and E.D Penney, *Calculus and Analytics Geometry*, Prentice Hall, Inc. 1988
- 6. E. W. Swokowski, *Calculus with Analytic Geometry*, PWS Publishers, Boston, Massachusetts, 1983.
- 7. M. Liebeck, A Concise introduction to pure Mathematics, CRC Press, 2011.
- 8. A. Kaseberg, Intermediate Algebra, Thomson Brooks/COLE, 2004.
- 9. J. Stewart, Calculus early transcendentals, 7th Edition, Brooks/COLE, 2008.

	Course Code:
Course Name: Algebra I (Group Theory)	
Course Structure:	Credit Hours: 03
<b>Prerequisites:</b> Elements of Set Theory and Mathematical	l Logic
Specific Objectives of course:	
This course introduces basic concepts of groups and their homomorphisms. The	
main objective of this course is to prepare students for courses which require a	
good back ground in group theory like Rings and Module	s, Linear Algebra, Group
Representation, Galois Theory etc.	
<u>Course Outline:</u>	
<b>Groups:</b> Definition of a group, subgroup, subgroup generated by a set. The cyclic groups, cosets and Lagrange's theorem. Normalizer centralizer. The center of a group. Equivalence relation in a group, conjugacy classes. Normal subgroups, quotient group	
<b>Group homomorphisms:</b> Homomorphisms and isomorphism and	
Automorphism. Kernel and image of homomorphism. Isomorphism theorems.	
Permutation groups. The cyclic decomposition of a permutation group. Cayley's	
theorem. Direct product of two groups and examples.	
Recommended Books:	
1. J. Rose, A Course on Group Theory, Cambridge U	niversity Press, 1978.
2. I. N. Herstein, <i>Topics in Algebra</i> , Xerox Publishing Company, 1964.	
3. P. M. Cohn, <i>Algebra</i> , John Wiley and Sons, London, 1974.	
4. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, BasicAbstract Algebra,	
Cambridge University Press, 1986.	
5. J. B. Fraleigh, A First Course in Abstract Algebra, Addison-Wesley	
Publishing Company, 2002.	
6. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.	
7. D. S. Dummit and R. M. Foote, Abstract Algebra, 3rd Edition, Addison-	
Wesley Publishing Company, 2004.	

Course Name: Linear Algebra	Course Code: MBS-441
Course Structure: Lectures: 3	Credit Hours: 3+1

Prerequisites: Calculus-I

## **Specific Objectives of course:**

Linear algebra is the study of vector spaces and linear transformations. The main objective of this course is to help students learn in rigorous manner, the tools and methods essential for studying the solution spaces of problems in mathematics, engineering, the natural sciences, and social sciences and develop mathematical skills needed to apply these to the problems arising within their field of study; and to various real world problems.

## **Course Outline:**

System of Linear Equations: Representation in matrix form. Matrices.

Operations on matrices. Echelon and reduced echelon form. Inverse of a matrix (by elementary row operations). Solution of linear system. Gauss-Jordan method. Gaussian elimination.

**Determinants:** Permutations of order two and three and definitions of determinants of the same order. Computing of determinants. Definition of higher order determinants. Properties. Expansion of determinants.

**Vector Spaces:** Definition and examples, subspaces. Linear combination and spanning set. Linearly Independent sets. Finitely generated vector spaces. Bases and dimension of a vector space. Operations on subspaces, Intersections, sums and direct sums of subspaces. Quotient Spaces.

**Linear mappings:** Definition and examples. Kernel and image of a linear mapping. Rank and nullity. Reflections, projections, and homotheties. Change of basis. Eigen-values and eigenvectors. Theorem of Hamilton-Cayley.

**Inner product Spaces:** Definition and examples. Properties, Projection. Cauchy inequality. Orthogonal and orthonormal basis. Gram Schmidt Process.

#### Diagonalization. Recommended Books:

- 1. Ch. W. Curtis, *Linear Algebra*, Springer 2004.
- 2. T. Apostol, *Multi Variable Calculus and Linear Algebra*, 2<sup>nd</sup> ed., John Wiley and sons, 1997.
- 3. H. Anton, C. Rorres, *Elementary Linear Algebra: Applications Version*, 10<sup>th</sup> Edition, John Wiley and sons, 2010.
- 4. S. Friedberg, A. Insel, Linear Algebra, 4<sup>th</sup> Edition,Pearson Education Canada, 2003.
- 5. S. I. Grossman, *Elementary Linear Algebra*, 5<sup>th</sup> Edition, Cengage Learning, 2004.

Course Name: Affine and Euclidean Geometry	Course Code: MBS-442
Course Structure: Lectures: 3	<b>Credit Hours:</b> 3

**Prerequisites: Calculus-I** 

Specific Objectives of course:

To familiarize mathematics students with the axiomatic approach to geometry from a logical, historical, and pedagogical point of view and introduce them with the basic concepts of Affine Geometry, Affine spaces and Platonic Ployhedra.

Course Outline:

**Vector spaces and affine geometry:** Collinearity of three points, ratio AB/BC. Linear combinations and linear dependent set versus affine combinations and affine dependent sets. Classical theorems in affine geometry: Thales, Menelaus, Ceva, Desargues. Affine subspaces, affine maps. Dimension of a linear subspace and of an affine subspace. **Euclidean geometry**: Scalar product, Cauchy-Schwartz inequality: norm of a vector, distance between two points, angles between two non-zero vectors. Pythagoras theorem, parallelogram law, cosine and sine rules. Elementary geometric loci. **Orthogonal transformations:** Isometries of plane (four types), Isometries of space (six types). Orthogonal bases.

**Platonic polyhedra**: Euler theorem on finite planar graphs. Classification of regular polyhedra in space. Isometries of regular polygons and regular polyhedra.

- 1. E. Rees, Notes on Geometry, Springer, 2004.
- 2. M. A. Armstrong, Groups and Symmetry, Springer, 1998.
- 3. H. Eves, *Fundamentals of Modern Elementary Geometry*, Jones and Bartlett Publishers International, 1992
- 4. S. Stahl, *The Poincare Half-Plane A Gateway to Modern Geometry*, Jones and Bartlett Publishers International, 1993.

Course Name: Discrete Mathematics	Course Code: MBS-444
Course Structure: Lectures: 3	<b>Credit Hours:</b> 3
Prerequisites: Mathematics at intermediate level	
Specific Objectives of course:	
Discrete Mathematics is study of distinct, un-related topics of mathematics; it embraces	
topics from early stages of mathematical development and recent additions to the	
discipline as well. The present course restricts only to counting methods, relations and	
graphs. The objective of the course is to inculcate in the students the skills that are	
necessary for decision making in non-continuous situations.	
Course Outline:	
Counting methods: Basic methods: product, inclusion-exclu	sion formulae.
Permutations and combinations. Recurrence relations and their solutions. Generating	
functions. Double counting. Applications. Pigeonhole principle, applications.	
Relations: Binary relations, n-ary Relations. Closures of relations. Composition of	
relations, inverse relation.	·
Graphs: Graph terminology. Representation of graphs. Grap	hs isomorphism. Algebraic
methods: the incidence matrix. Connectivity, Eulerian and Ha	miltonian paths. Shortest

# path problem. Trees and spanning trees. Complete graphs and bivalent graphs. **Recommended Books:**

- 1. B. Bollobas, *Graph Theory*, Springer Verlag, New York, 1979.
- 2. K.R. Parthasarathy, Basic Graph Theory, McGraw-Hill, 1994
- 3. K.H. Rosen, *Discrete Mathematics and its Application*, McGraw-Hill, 6<sup>th</sup> edition, 2007.
- 4. B. Kolman, R.C. Busby, S.C. Ross, *DiscreteMathematical Structures*, Prentice-Hall of India, New Delhi, 5<sup>th</sup> edition, 2008.
- 5. A. Tucker, *Applied Combinatorics*, John Wiley and Sons, Inc New York, 2002.
- 6. R. Diestel, *Graph Theory*, 4<sup>th</sup> edition, Springer- Verlag, New York, 2010.
- 7. N.L. Brigs, *Discrete Mathematics*, Oxford University Press, 2003
- 8. K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall,

Course Name: Topology	Course Code: MBS-551
Course Structure: Lectures: 3	Credit Hours: 3
Prerequisites: Calculus-I	

#### **Specific Objectives of course:**

The aim of this course is to introduce the students to metric spaces and topological spaces. After completion of this course, they would be familiar with separation axioms, compactness and completeness. They would be able to determine whether a function defined on a metric or topological space is continuous or not and what homeomorphisms are.

#### Course Outline:

Topological spaces: Examples; open and closed subsets, metric spaces, neighbourhoods. Examples. Limit points and accumulation points. Interior, closure, dense subsets. Constructing new topological spaces: Cartesian products, induced topology and quotient topology. Continuous maps, open and closed maps, homeomorphisms. Examples: R, RxR, S^1, S^2, torus, cylinder. Cauchy sequences, complete metric spaces. Separation axioms. Compact spaces. Properties. Power of Compactness. Image of a compact set through a continuous map. Compactness and completeness of metric spaces. Connected spaces, connected components. Properties. Image of a connected set through a continuous map. Path-connectedness.

#### **Recommended Books:**

- 1. J. Kelly, *General Topology*, Springer, 2005.
- 2. K. Janich, *Topology*, Springer, 1994.
- 3. J. Hocking, G. Young, *Topology*, Dover Publications, 1961.
- 4. J. R. Munkres, *Topology A First Course*, Prentice-Hall, 2003.
- 5. G. Simmons, Topology and modern analysis, McGraw-Hill, 1963.
- 6. S. Lipschutz, General Topology, McGraw-Hill, 2004.
- 7. J. Dugundji, *Topology*, Allyn and Bacon, 1966.

Course Name: Differential Geometry and TensorCourse Code: MBS-552

Analysis	
Course Structure: Lectures: 4	Credit Hours: 04

Prerequisites: Calculus-I

**Specific Objectives of course:** 

After having completed this course, the students would be expected to understand classical concepts in the local theory of curves and surfaces including normal, principal, mean, curvature, and geodesics. They will also learn about tensors of different ranks.

# **Course Outline:**

**Theory of Space Curves:** Introduction, index notation and summation convention. Space curves, arc length, tangent, normal and binormal. Osculating, normal and rectifying planes. Curvature and torsion. The Frenet-Serret theorem. Natural equation of a curve. Involutes and evolutes, helices. Fundamental existence theorem of space curves.

**Theory of Surfaces:** Coordinate transformation. Tangent plane and surface normal. The first fundamental form and the metric tensor. The second fundamental form. Principal, Gaussian, mean, geodesic and normal curvatures. Gauss and Weingarten equations. Gauss andCodazzi equations.

**Tensor Analysis:** Einstein summation convention. Tensors of different ranks. Contravariant, covariant and mixed tensors. Addition, subtraction, inner and outer products of tensors. Contraction theorem, quotient law. The line element and metric tensor. Christoffel symbols.

- 1. R. S. Millman and G. D. Parker, *Elements of Differential Geometry*, Prentice-Hall, New Jersey, 1977.
- 2. A. Goetz, Introduction to Differential Geometry, Addison-Wesley, 1970.
- 3. E. Kreyzig, Differential Geometry, Dover, 1991.
- 4. M. M. Lipschutz, *Schaum's Outline of Differential Geometry*, McGraw Hill, 1969.
- 5. D. Somasundaram, *Differential Geometry*, Narosa PublishingHouse, New Delhi. 2005.
- 6. M. R. Spiegel, Vector Analysis, McGraw Hill BookCompany, Singapore, 1981.
- 7. A. W. Joshi, *Matrices and Tensors in Physics*, Wiley EasternLimited, 1991.
- 8. F. Chorlton, Vector and Tensor Methods, Ellis HorwoodPublisher, U.K., 1977.

Course Name: Ordinary Differential Equations	<b>Course Code:</b> MBS-553
Course Structure: Lectures: 3	<b>Credit Hours:</b> 3
Prerequisites: Calculus-I	
Specific Objectives of course:	
To introduce students to the formulation, classification of differential equations	
and existence and uniqueness of solutions. To provide skill in solving initial value	
and boundary value problems. To develop understanding and skill in solving first	
and second order linear homogeneous and nonhomogeneous differential equations	
and solving differential equations using power series methods.	

# **Course Outline:**

**Preliminaries**: Introduction and formulation, classification of differential equations, existence and uniqueness of solutions, introduction of initial value and boundary value problems

**First order ordinary differential equations:** Basic concepts, formation and solution of differential equations. Separable variables, Exact Equations,

Homogeneous Equations, Linear equations, integrating factors. Some nonlinear first order equations with known solution, differential equations of Bernoulli and Ricaati type, Clairaut equation, modeling with first-order ODEs, Basic theory of systems of first order linear equations, Homogeneous linear system with constant coefficients, Non homogeneous linear system

Second and higher order linear differential equations: Initial value and boundary value problems, Homogeneous and non-homogeneous equations, Superposition principle, homogeneous equations with constant coefficients, Linear independence and Wronskian, Nonhomogeneous equations, undetermined coefficients method, variation of parameters, Cauchy-Euler equation, Modeling. Sturm-Liouville problems: Introduction to eigen value problem, adjoint and self adjoint operators, self adjoint differential equations, eigen values and eigen functions, Sturm-Liouville (S-L) boundary value problems, regular and singular S-L problems, properties of regular S-L problems

**Series Solutions:** Power series, ordinary and singular points, Existence of power series solutions, power series solutions, types of singular points, Frobenius theorem, Existence of Frobenius series solutions, solutions about singular points, The Bessel, modified Bessel, Legendre and Hermite equations and their solutions.

# **Recommended Books:**

- 1. Dennis G. Zill and Michael R., Differential equations with boundary-value problems by Cullin 5<sup>th</sup> Edition Brooks/Cole, 1997.
- 2. William E. Boyce and Richard C. Diprima, Elementary differential equations and boundary value problems, Seventh Edition John Wiley & Sons, Inc
- 3. V. I. Arnold, Ordinary Differential Equations, Springer, 1991.
- 4. T. Apostol, *Multi Variable Calculus and Linear Algebra*, 2nd ed., John Wiley and sons, 1997.

Course Name: Real Analysis-I	Course Code:MBS-554
Course Structure: Lectures: 3	<b>Credit Hours:</b> 3
Prerequisites: Calculus-III	

## **Specific Objectives of course:**

This is the first course in analysis. It develops the fundamental ideas of analysis and is aimed at developing the students' ability in reading and writing mathematical proofs. Another objective is to provide sound understanding of the axiomatic foundations of the real number system, in particular the notions of completeness and compactness.

## **Course Outline:**

Number Systems: Ordered fields. Rational, real and complex numbers.

Archimedean property, supremum, infimum and completeness.

**Topology of real numbers**: Convergence, completeness, completion of real numbers. Open sets, closed sets, compact sets. Heine Borel Theorem. Connected sets.

**Sequences and Series of Real Numbers:** Limits of sequences, algebra of limits. Bolzano Weierstrass Theorem. Cauchy sequences, liminf, limsup. Limits of series, convergences tests, absolute and conditional convergence. Power series.

**Continuity:** Functions, continuity and compactness, existence of minimizers and maximizers, uniform continuity. Continuity and connectedness, Intermediate mean Value Theorem. Monotone functions and discontinuities.

Differentiation: Mean Value Theorem, L'Hopital's Rule, Taylor's Theorem.

# **Recommended Books:**

- 1. S. Lang, Analysis *I*, Addison-Wesley Publ. Co., Reading, Massachusetts, 1968.
- 2. W. Rudin, *Principles of Mathematical Analysis*, 3rd ed., Mc.Graw Hill, 1976.
- 3. B. S. Thomson, J. B. Bruckner and A. M. Bruckner, *ElementaryReal Analysis*, 2nd Ed. 2008.
- 4. G. Boros, V. Moll, Irresistible Integrals: Symbolics, Analysisan Experiments in the Evaluation of Integrals, Cambridge University Press, 2004.
- 5. J. Borwein, D. Bailey, R. Girgenson, *Experimentation inMathematics: Computational Paths to discovery*, Wellesley, MA, A.K. Peters, 2004.
- 6. G. Bartle, R. Sherbert, *Introduction to Real Analysis*, 3<sup>rd</sup> edition, John Wiley, New York, 1999.

Course Name: Algebra-II (Ring Theory)	Course Code: MBS-555
Course Structure: Lectures: 3	<b>Credit Hours:</b> 3

## Prerequisites: Algebra-I

# **Specific Objectives of course:**

This is a course in advanced abstract algebra, which builds on the concepts learnt in Algebra I. The objectives of the course are to introduce students to the basic ideas and methods of modern algebra and enable them to understand the idea of a ring and an integral domain, and be aware of examples of these structures in mathematics; appreciate and be able to prove the basic results of ring theory; appreciate the significance of unique factorization in rings and integral domains.

# Course Outline:

**Rings:** Definition, examples. Quadratic integer rings. Examples of noncommutative rings. The Hamilton quaternions. Polynomial rings. Matrix rings. Units, zero-divisors, nilpotents, idempotents. Subrings, Ideals. Maximal and prime Ideals. Left, right and two-sided ideals; Operations with ideals. The ideal generated by a set. Quotient rings. Ring homomorphism. The isomorphism theorems, applications. Finitely generated ideals. Rings of fractions. **Integral Domain:** The Chinese remainder theorem. Divisibility inintegral domains, greatest common divisor, least common multiple. Euclidean domains. The Euclidean algorithm. Principal ideal domains. Prime and irreducible elements in an integral domain. Gauss lemma, irreducibility criteria for polynomials. Unique factorization domains.

Finite fields. Polynomials in several variables. Symmetric polynomials. The fundamental theorem of symmetric polynomials.

# **Recommended Books:**

- 1. J. Rose, A Course on Group Theory, Cambridge University Press, 1978.
- 2. I. N. Herstein, Topics in Algebra, Xerox Publishing Company, 1964.
- 3. P. M. Cohn, Algebra, John Wiley and Sons, London, 1974.
- 4. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, *BasicAbstract Algebra*, Cambridge University Press, 1986.
- 5. J. B. Fraleigh, *A First Course in Abstract Algebra*, Addison-Wesley Publishing Company, 2002.
- 6. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
- 7. D. S. Dummit and R. M. Foote, *Abstract Algebra*, 3<sup>rd</sup> Edition, Addison-Wesley Publishing Company, 2004.
- 8. D. S. Dummit and R. M. Foote, *Abstract Algebra*, 3<sup>rd</sup> Edition, Addison-Wesley Publishing Company, 2004.

Course Name: Classical Mechanics	Course Code: MBS-561
Course Structure: Lectures: 3	Credit Hours: 3
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# Specific Objectives of course:

To provide solid understanding of classical mechanics and enable the students to use this understanding while studying courses on quantum mechanics, statistical mechanics, electromagnetism, fluid dynamics, space-flight dynamics, astrodynamics and continuum mechanics

astrodynamics and continuum mechanics.

# **Course Outline:**

**Kinematics:** Rectilinear motion of particles. Uniform rectilinear motion, uniformly accelerated rectilinear motion. Curvilinear motion of particle, rectangular components of velocity and acceleration. Tangential and normal components. Radial and transverse components. Projectile motion.

**Kinetics:** Work, power, kinetic energy, conservative force fields. Conservation of energy, impulse, torque. Conservation of linear and angular momentum. Non-conservative forces.

**Simple Harmonic Motion:** The simple harmonic oscillator, period, frequency. Resonance and energy. The damped harmonic oscillator, over damped, critically damped and under damped. Motion, forces and vibrations.

**Central Forces and Planetary Motion:** Central force fields, equations of motion, potential energy, orbits. Kepler's law of planetary motion. Apsides and apsidal angles for nearly circular orbits. Motion in an inverse square field.

**Planer Motion of Rigid Bodies:** Introduction to rigid and elastic bodies, degree of freedom, translations, rotations, instantaneous axis and center of rotation, motion of the center of mass. Euler's theorem and Chasles' theorem. Rotation of a rigid body about a fixed axis, moments and products of inertia. Parallel and

perpendicular axis theorem.

Motion of Rigid Bodies in Three Dimensions: General motion of rigid bodies in space. The momental ellipsoid and equimomental systems. Angular momentum vector and rotational kinetic energy. Principal axes and principal moments of inertia. Determination of principal axes by diagonalizing the inertia matrix.

**Euler Equations of Motion of a Rigid Body:** Force free motion. Free rotation of a rigid body with an axis of symmetry. Free rotation of a rigid body with three different principal moments. The Eulerian angles, angular velocity and kinetic energy in terms of Euler angles. Motion of a spinning top and gyroscopes-steady precession, sleeping top.

# **Recommended Books:**

- 1. E. DiBenedetto, *Classical Mechanics. Theory and Mathematical Modeling*, ISBN: 978-0-8176-4526-7,Birkhauser Boston, 2011.
- 2. John R. Taylor, *Classical Mechanics*, ISBN: 978-1-891389-22-1, University of Colorado, 2005.
- 3. H. Goldstein, *Classical Mechanics*, Addison-Wesley PublishingCo., 1980.
- 4. C. F. Chorlton, Text Book of Dynamics, Ellis Horwood, 1983.
- 5. M. R. Spiegel, *Theoretical Mechanics*, 3<sup>rd</sup> Edition, Addison-Wesley Publishing Company, 2004.
- 6. G. R. Fowles and G. L. Cassiday, *Analytical Mechanics*, 7<sup>th</sup>edition, Thomson Brooks/COLE, USA, 2005.

Course Name: Complex Analysis	Course Code: MBS-562
Course Structure: Lectures: 3	<b>Credit Hours:</b> 3

#### Prerequisites: Analysis-I

## **Specific Objectives of course:**

This is an introductory course in complex analysis, giving the basics of the theory along with applications, with an emphasis on applications of complex analysis and especially conformal mappings. Students should have a background in real analysis (as in the course Real Analysis I), including the ability to write a simple proof in an analysis context.

# **Course Outline:**

**Introduction**: The algebra of complex numbers, Geometric representation of complex numbers, Powers and roots of complex numbers.

**Functions of Complex Variables:** Definition, limit and continuity, Branches of functions, Differentiable and analytic functions. The Cauchy-Riemann equations, Entire functions, Harmonic functions, Elementary functions: The exponential, Trigonometric, Hyperbolic, Logarithmic and Inverse elementary functions, Open mapping theorem. Maximum modulus theorem.

**Complex Integrals:** Contours and contour integrals, Cauchy-Goursat theorem, Cauchy integral formula, Lioville's theorem, Morerea's theorem.

**Series:** Power series, Radius of convergence and analyticity, Taylor's and Laurent's series, Integration and differentiation of power series. Singularities, Poles and residues: Zero, singularities, Poles and Residues, Types of singular

points, Calculus of residues, contour integration, Cauchy's residue theorem with applications. Mobius transforms, Conformal mappings and transformations.

# **Recommended Books:**

- 1. R. V. Churchill, J. W. Brown, *Complex Variables and Applications*, 5<sup>th</sup> edition, McGraw Hill, New York, 1989.
- 2. J. H. Mathews and R. W. Howell, *Complex Analysis forMathematics and Engineering*, 2006.
- 3. S. Lang, Complex Analysis, Springer-Verlag, 1999.
- 4. R. Remmert, *Theory of Complex Functions*, Springer-Verlag, 1991.
- 5. W. Rudin, Real and Complex Analysis, McGraw-Hill, 1987.

Course Name: Partial Differential Equations	Course Code:
Course Structure: Lectures: 3	Credit Hours: 3
Prerequisites: Ordinary Differential Equations	

# Specific Objectives of course:

Partial Differential Equations (PDEs) are at the heart of applied mathematics and many other scientific disciplines. The course aims at developing understanding about fundamental concepts of PDEs theory, identification and classification of their different types, how they arise in applications, and analytical methods for solving them. Special emphasis would be on wave, heat and Laplace equations. **Course Outline:** 

**First order PDEs:** Introduction, formation of PDEs, solutions of PDEs of first order, The Cauchy's problem for quasilinear first order PDEs, First order nonlinear equations, Special types of first order equations

**Second order PDEs:** Basic concepts and definitions, Mathematical problems, Linear operators, Superposition, Mathematical models: The classical equations, the vibrating string, the vibrating membrane, conduction of heat solids, canonical forms and variable, PDEs of second order in two independent variables with constant and variable coefficients, Cauchy's problem for second order PDEs in two independent variables

**Methods of separation of variables:** Solutions of elliptic, parabolic and hyperbolic PDEs in Cartesian and cylindrical coordinates

**Laplace transform:** Introduction and properties of Laplace transform, transforms of elementary functions, periodic functions, error function and Dirac delta function, inverse Laplace transform, convolution theorem, solution of PDEs by Laplace transform, Diffusion and wave equations

**Fourier transforms**: Fourier integral representation, Fourier sine and cosine representation, Fourier transform pair, transform of elementary functions and Dirac delta function, finite Fourier transforms, solutions of heat, wave and Laplace equations by Fourier transforms.

- 1. Myint UT, *Partial Differential Equations for Scientists and Engineers*, 3<sup>rd</sup> edition, North Holland, Amsterdam, 1987.
- 2. Dennis G. Zill, Michael R. Cullen, *Differential equations with boundary value problems*, Brooks Cole, 2008.

- 3. John Polking, Al Boggess, *Differential Equations with Boundary Value Problems*, 2<sup>nd</sup> Edition, Pearson, July 28, 2005.
- 4. J. Wloka, *Partial Differential Equations*, Cambridge Universitypress, 1987.

Course Name: Real Analysis-II	Course Code: MBS-564
Course Structure: Lectures: 3	<b>Credit Hours:</b> 3
<b>Prerequisites:</b> Real Analysis-I	

A continuation of Real Analysis-I, this course will continue to cover the fundamentals of real analysis, concentrating on the Riemann Stieltjes integrals, Functions of Bounded Variation, Improper Integrals, and convergence of series. Emphasis would be on proofs of main results.

## **Course Outline:**

**The Riemann-Stieltjes Integrals:** Definition and existence of integrals. Properties of integrals. Fundamental theorem of calculus and its applications. Change of variable theorem. Integration by parts.

**Functions of Bounded Variation:** Definition and examples. Properties of functions of bounded variation.

**Improper Integrals:** Types of improper integrals, tests for\ convergence of improper integrals. Beta and gamma functions. Absolute and conditional convergence of improper integrals.

**Sequences and Series of Functions:** Power series, definition of point-wise and uniform convergence. Uniform convergence and continuity. Uniform convergence and differentiation. Examples of uniform convergence.

- 1. S. Lang, *Analysis I, II*, Addison-Wesley Publ. Co., Reading, Massachusetts, 1968, 1969.
- 2. W. Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw-Hill, 1976.
- 3. K. R. Davidson and A. P. Donsig, *Real Analysis withReal Applications*, Prentice Hall Inc., Upper Saddle River, 2002.
- 4. G. B. Folland, *Real Analysis*, 2<sup>nd</sup> Edition, John Wiley and Sons, New York, 1999.
- 5. E. Hewitt and K. Stromberg, *Real and Abstract Analysis*, Springer-Verlag, Berlin Heidelberg New York, 1965.
- 6. H. L. Royden, *Real Analysis*, 3<sup>rd</sup> Edition, Macmillan, New York, 1988.
- 7. G. Bartle, R. Sherbert, *Introduction to Real Analysis*, 3<sup>rd</sup> edition, John Wiley, New York, 1999.

Course Name: Functional Analysis	Course Code: MBS-565
Course Structure: Lectures: 3	Credit Hours: 3
Prerequisites: Analysis I	
Specific Objectives of course: This course extends methods of linear algebra and	
analysis to spaces of functions, in which the interaction between algebra and	

analysis allows powerful methods to be developed. The course will be mathematically sophisticated and will use ideas both from linear algebra and analysis.

# **Course Outline:**

**Metric Space**: Review of metric spaces, Convergence in metric spaces, Complete metric spaces, Dense sets and separable spaces, No-where dense sets, Baire category theorem.

**Normed Spaces:** Normed linear spaces, Banach spaces, Equivalent norms, Linear operator, Finite dimensional normed spaces, Continuous and bounded linear operators, Dual spaces.

**Inner Product Spaces:** Definition and examples, Orthonormal sets and bases, Annihilators, projections, Linear functionals on Hilbert spaces. Reflexivity of Hilbert spaces.

- 1. A. V. Balakrishnan, *Applied Functional Analysis*, 2<sup>nd</sup> edition, Springer-Verlag, Berlin, 1981.
- 2. J. B. Conway, A Course in Functional Analysis, 2<sup>nd</sup> ed., Springer-Verlag, Berlin, 1997.
- 3. K. Yosida, Functional Analysis, 5th ed., Springer-Verlag, Berlin, 1995.
- 4. E. Kreyszig, *Introduction to Functional Analysis withApplications*, John Wiley and Sons, 2004.

Course Name: Numerical Analysis-I	Course Code: MBS-671
Course Structure: Lectures: 3, Practical: 1	<b>Credit Hours:</b> 3+1
Prerequisites: Calculus-I, Linear Algebra	

This course is designed to teach the students about numerical methods and their theoretical bases. The course aims at inculcating in the students the skill to apply various techniques in numerical analysis, understand and do calculations about errors that can occur in numerical methods and understand and be able to use the basics of matrix analysis.

#### **Course Outline:**

Error analysis: Floating point arithmetic, approximations and errors.

**Methods for the solution of nonlinear equations**: Bisection method, regula-falsi method, fixed point iteration method, Newton-Raphson method, secant method, error analysis for iterative methods.

**Interpolation and polynomial approximation**: Lagrange interpolation, Newton's divided difference formula, forward, backward and centered difference formulae, interpolation with a cubic spline, Hermite interpolation, least squares approximation. **Numerical differentiation**: Forward, backward and central difference formulae, Richardson's extrapolation.

**Numerical integration**: Rectangular rule, trapezoidal rule, Simpson's 1/3 and 3/8 rules, Boole's and Weddle's rules, Newton-Cotes formulae, Gaussian quadrature.

**Numerical solution of a system of linear equations**: Direct methods: Gaussian elimination method, Gauss-Jordan method; matrix inversion; LU-factorization; Doolittle's, Crout's and Cholesky's methods, Iterative methods: Jacobi, Gauss-Seidel and SOR. The use of software packages/programming languages for above mentioned topics is recommended.inverse square field.

- 1. C.F. Gerald and P.O.Wheatley, Applied Numerical Analysis, Pearson Education, Singapore, 2005.
- 2. R. L. Burden and J. D. Faires: Numerical Analysis, latest edition, PWS Pub. Co.
- 3. J.H. Mathews, Numerical Methods for Mathematics, latest Edition, Prentice Hall International.
- 4. S. C. Chapra and R. P. Canale: Numerical Methods for Engineers, 6<sup>th</sup> edition, McGraw Hill.
- 5. W. E. Boyce, R. C. DiPrima, Elementary Differential Equations and Boundary Value Problems, John Wiley \& Sons, Inc., 2001.
- 6. L. Debnath, Nonlinear Partial Differential Equations for Scientists and Engineers, Birkhauser-Boston, 2005.
- 7. Alexander Komech, Andrew Komech, Principles of Partial Differential Equations, Springer-New York, 2009.
- 8. H. Richard, Elementary Applied Partial Differential Equations, Prentice-Hall International, Inc., London 1987.
- 9. Weinberger, Hans F., A First Course in Partial Differential Equations with Complex Variables and Transform Methods, Dover Publications, Inc., 1995.
- 10. R. Kent Nagle, Edward B. Saff, Arthur David Snider, Fundamentals of Differential Equations, Addison Wesley Longman, Inc., 2000.

Course Name: Number Theory	Course Code: MBS-672
Course Structure: Lectures: 3	Credit Hours: 3
Prerequisites: Linear Algebra	

The focus of the course is on study of the fundamental properties of integers and develops ability to prove basic theorems. The specific objectives include study of division algorithm, prime numbers and their distributions, Diophantine equations, and the theory of congruences.

## **Course Outline:**

**Preliminaries:** Well-ordering principle. Principle of finite induction.

**Divisibility theory:** The division algorithms. Basis representation theorem. Prime and composite numbers. Canonical decomposition. The greatest common divisor. The Euclidean algorithm. The fundamental theorem of arithmetic. Least common multiple.

**Linear Diophantine equations:** Congruences. Linear congruences. System of linear congruences. The Chinese remainder theorem. Divisibility tests. Solving polynomial congruences. Fermat's and Euler's theorems. Wilson's theorem.

**Arithmetic functions:** Euler's phi-function. The functions of J and sigma. The Mobius function. The sieve of Eratosthenes. Perfect numbers. Fermat and Mersenne primes.

**Primitive Roots and Indices:** The order of an integer mod n. Primitive roots for primes. Composite numbers having primitive roots.

**Quadratic residues:** Legendre symbols and its properties. The quadratic reciprocity law. Quadratic congruences with composite moduli. Pythagorean triples.

Representing numbers as sum of two squares.

- 1. D.M. Burton, *Elementary Number Theory*, McGraw-Hill, 2007.
- 2. W.J. Leveque, *Topics in Number Theory*, vols. I and II, Addison- Wesley, 1956.
- 3. S.B. Malik, *Basic Number Theory*, Vikas Publishing house, 1995.
- 4. K.H. Rosen, *Elementary Number Theory and its Applications*, 5<sup>th</sup> edition, Addison Wesley, 2005.
- 5. I. Niven, H.S. Zuckerman, H.L. Montgomery, *An Introduction to the theory of Numbers*, John Wiley and Sons, 1991.
- 6. A. Adler, J.E. Coury, *The Theory of Numbers*, Jones and Bartlett Publishers, 1995

Course Name: Mathematical Methods	Course Code: MBS-676
Course Structure: Lectures: 3	Credit Hours: 3
Prerequisites: Calculus-III	

The main objective of this course is to provide the students with a range of mathematical methods that are essential to the solution of advanced problems encountered in the fields of applied physics and engineering. In addition this course is intended to prepare the students with mathematical tools and techniques that are required in advanced courses offered in the applied physics and engineering programs.

## **Course Outline:**

**Fourier Methods:** The Fourier transforms. Fourier analysis of the generalized functions. The Laplace transforms. Hankel transforms for the solution of PDEs and their application to boundary value problems.

**Green's Functions and Transform Methods:** Expansion for Green's functions. Transform methods. Closed form Green's functions.

**Perturbation Techniques:** Perturbation methods for algebraic equations. Perturbation methods for differential equations.

**Variational Methods:** Euler-Lagrange equations. Integrand involving one, two, three and n variables. Special cases of Euler-Lagrange's equations. Necessary conditions for existence of an extremum of a functional. Constrained maxima and minima.

- 1. D. L. Powers, *Boundary Value Problems and PartialDifferential Equations*, 5<sup>th</sup> edition, Academic Press, 2005.
- 2. W. E. Boyce, *Elementary Differential Equations*, 8<sup>th</sup> edition, John Wiley and Sons, 2005.
- 3. M. L. Krasnov, G. I. Makarenko and A. I. Kiselev, *Problemsand Exercises in the Calculus of Variations*, Imported Publications, Inc., 1985.
- 4. J. W. Brown and R. V. Churchil, Fourier Series and BoundaryValue Problems, McGraw Hill, 2006.
- **5.** A. D. Snider, *Partial Differential Equations: Sources andSolutions*, Prentice Hall Inc., 1999.

Course Name: Probability TheoryCourse Code: MBS-681
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Course Structure: Lectures: 3	Credit Hours: 3	
Prerequisites: Statistics	· ·	
Specific Objectives of course:		
A prime objective of the course is to introduce the student	s to the fundamentals of	
probability theory and present techniques and basic results of the theory and illustrate		
these concepts with applications. This course will also present the basic principles of		
random variables and random processes needed in 24 applications.		
Course Outline:		
Finite probability spaces: Basic concept, probability and	related frequency,	
combination of events, examples, Independence, Random variables, Expected value.		
Standard deviation and Chebyshev's inequality. Independence of random variables.		
Multiplicativity of the expected value. Additivity of the variance, Discrete probability		
distribution.		
Probability as a continuous set function: sigma-algebras, examples. Continuous		
random variables, Expectation and variance. Normal random variables and continuous		
probability distribution.		
<b>Applications:</b> de Moivre-Laplace limit theorem, weak and strong law of large numbers.		
The central limit theorem, Markov chains and continuous Markov process.		
Recommended Books:		
1. M. Capinski, E. Kopp, <i>Measure, Integral and Probability</i> , Springer-Verlag,		
1998. 2 D. M. Dudlau, Beel Angeleric and Buck skility. Combridge University Daves 2004		
2. K. M. Dudley, <i>Real Analysis and Probability</i> , Cambridge University Press, 2004.		
5. 5. 1. Resilick, A Probability Pain, Dirkhauser, 1999	7. d. Prontico Holl 1008	
<ol> <li>S. KUSS, A just Course in Froodouly Theory, 5 ed., Fiendle Hall, 1998.</li> <li>Debort P. Ash. Rasia Probability Theory, Dovor, P. 2008.</li> </ol>		

5. Robert B. Ash, *Basic Probability Theory*, Dover. B, 2008.

Course Name: Optimazation theory	Course Code:
Course Structure: Lectures: 3	<b>Credit Hours:</b> 3
Prerequisites:	

The subject of optimization can be studied as a branch of pure mathematics and has application in nearly all the branches of science and technology. Therefore this course aims to equip students from those aspects of optimization methods which are of importance in real life problem solving.

#### **Course Outline:**

#### **Basic Results:**

Definition, Condition for unconstrained variables, Equality constraints, General consideration and necessary conditions of Inequality constraints. Convexity, abnormal point and sufficient conditions for Inequality. Sadle point condition and Duality.

#### **Unconstrained Optimization:**

Line search Methods, General search methods, Gradient Methods, Newton and Quasi Newton Methods.

#### Linear Programming:

Solution of LP Problem, Duality.

#### **Constrained Optimization:**

General Properties of the solution, Projection Methods, Quadratic Programming, Application of projection methods to nonlinear constraints.

#### **Recommended Books:**

D.M. Greig, Optimization, Longman London and New York, 1980.